J. Pure Appl. & Ind. Phys. Vol.3 (2), 172-177 (2013)

Jeans Instability of Rotating Viscous Molecular Cloud of Quantum Plasma under the Effect of Black-Body Radiation

R. K. PENSIA[1], V.KUMAR[2*], G. S. KACHHAWA[2], A. K. PATIDAR[2] and V. SHRIVASTAVA[2]

[1]Department of Physics,
Govt. Girls (P.G.) College, Neemuch, M. P., INDIA
[2]Department of Physics,
Pacific University, Udaipur, Raj., INDIA

(Received on: March 11, 2013)

ABSTRACT

The problem of Jeans instability of rotating viscous molecular cloud of quantum plasmas has been investigated as considering the optically thick medium incorporating effect of porosity, permeability and black body radiation. The equations of the problem are stated and dispersion relation is obtained using normal mode analysis. The criterions of instability are discussed as our interest.

Keywords: Quantum Plasma, Black Body Radiation, Rotation, Viscosity, Permeability, Jeans instability.

1. INTRODUCTION

The Physics of quantum plasmas is recently studied rather intensively because of its importance for a number of applications in space plasmas, the ISM structure as well as in the laboratory Pines[1,2]. The gravitational instability is one of the fundamental concepts of modern astrophysical plasma and it is connected with the fragmentation of ISM structure. Jeans[3] was the first to investigate the gravitational instability of infinite homogeneous matter with the help of classical equations of hydrodynamics and Newtonian gravitation theory. Jeans made the assumption that small perturbations in the density, velocity and gravitational field are superimposed on a stationary and uniform mass distribution when the problem

is formulated in this way, the pointed Jeans wavelength arises, showing the minimum spatial scale a perturbation must have such that the elastic forces of matter are no longer capable of withstanding the gravitational forces. In each volume element the density perturbation will follow the law

$$\frac{\delta\rho}{\rho} = A_1 e^{i\omega t} + A_2 e^{-i\omega t},$$

where $\omega = \sqrt{c^2k^2 - 4\pi\rho G}$, G is the gravitational constant, c is the velocity of sound, k^2 represents the square wave vector of periodic spatial perturbation, and the constant A1, A2 are set by the initial condition. Under the Jeans criterion of instability, that is, if

$$\lambda > \lambda_j = \left(\frac{\pi c^2}{G\rho}\right)^{1/2},$$

The quantity in the expression for $\sigma\,(i\omega)$ will be positive and $\frac{\delta\rho}{\rho}$ will depend exponentially on time, whereas if $\lambda < \lambda_J$ the quantity $\sigma\,(i\omega)$ will be imaginary and oscillations-acoustic waves will develop.

Studies of gravitational instability usually deal with single component material having a definite density and sonic velocity. When other components such as photon gas are included in studies based on the theory of big-bang universe, we generally find it is necessary to consider non-gravitational (dissipative) processes that would inhibit the growth of perturbations in ordinary matter. Attempts have also been made to allow for contribution of the photon gas toward the non-stationary general (background) gravitational field, which would affect the general course of expansion and they indirectly influence the development of ordinary matter perturbations. The quantum correction plays an important role in the stability analysis and quantum plasma was first investigated by Pines[1, 2]. The kinetic model of the quantum electro dynamical properties of non-thermal plasmas has been studied by Bezzerides and Du Bios[4]. The Covariant wigner function description of relativistic quantum plasmas is given by Hakim and Heyvaerts[5]. The Jeans instability of self-gravitating astrophysical quantum dusty plasma has been investigated by Shukla and Stenflo[6]. Masood *et al*.[7] extended the above problem by considering a multi-component self-gravitating quantum Bohm potential and statistical terms on electrons and ions were considered. The Jeans instability in homogeneous quantum dusty plasma in the presence of a magnetic field and quantum correction was examined by Salimullah *et al*.[8].

The coriolis force plays a crucial role in a variety of astrophysical and space environments often providing the dominant mechanism for plasma drift against the disturbances. Chandrashekhar[9] have discussed the effect of the coriolis force on problem of thermal instability and on the stability of a viscous flow in the presence of a magnetic field. Lehnert[10, 11] have studied the problem of magneto-turbulence and he suggested that this force is important for a large range of wave numbers of disturbances in the interior of the sun. The role of coliolis force and suspended particles in the fragmentation of matter in the central region of galaxy has been studied by Pensia *et al*.[12]. In this connection, many authors, Prajapati *et al*.[13], Patidar *et al*.[14], Pensia *et al*.[15] have discussed the effect of rotation on the

gravitational instability of a homogenous plasma considering the effects of various parameters Prajapati *et al*.[13], Patidar *et al*.[14], Pensia *et al*.[15].

However, another question of important concerns the problem of black body radiation interaction among density perturbations if the different components in a medium consisting of several different materials. Problems of this kind are of intrinsic interest, but beyond that they might turn out to be of central significance in light of the contribution of the gravitational contraction of matter during the pregalactic phase of the evolution of the universe. The role of heat-loss function has been discussed by many researchers as Pensia *et al*.[16], Bora and Talwar[17], Kaothekar and Chhajlani[18], Pensia *et al*.[19], Vranjes and Cadez [20].

From the above studies, we find that the quantum correction viscosity, rotation and black body radiation are the important parameters to discuss with the gravitational instability of plasma. Thus the present contribution intends to analyze the jeans instability of viscous infinite homogeneous rotating quantum plasmas of interstellar medium under the influence of black body radiation. Simple hydrodynamical model has been used to discuss about the stability and instability of interest with linear normal mode analysis of acoustic turbulence. To avoid the discussion about the additional convective instability influences which may occur in such a system if the temperature T is a decreasing function of a coordinate, therefore, we shall assume T to be constant. The radiation function is similar to those of the constant functions considered earlier by Vranjes and Cadez [20].

The purpose of this paper is to clarify the criterions of Jeans instability of infinite homogeneous viscous quantum plasma under the influence of corilolis force and black body radiations.

The paper is organized in the following fashion; we derive a set of fluid equation in the bulk frame suitable for system. The medium is taken as optically thick and black body radiation is assumed. To clarify the significance of above parameters, we suppose that only one direction of medium has been subjected to perturbation. The dispersion relations is obtained, which is modified due to the presence of these parameters. Our result will possess its own limit of instability and its own Jeans wavelength. What laws of gravitational instability will operate in such medium? What will be the number of unstable solutions (the number of real ω in the expression for $\delta\rho/\rho$ in each component), and which parameters of the fluid will determine them? These are the questions to be addressed in the paper.

2. EQUATIONS OF THE PROBLEM

Let us consider an infinite homogeneous, viscous and rotating self-gravitating molecular cloud of quantum plasma. The medium is taken to as optically thick and black body radiation is considered. We introduced the quantum effects through the Bohm potential term in the momentum transfer equation. For our simplicity, it is to avoid discussion about the additional convective instability phenomena which may occur in such a system if the temperature T is a decreasing function of a coordinate, we shall assume T to be constant.

The momentum transfer equation for magnetized quantum plasma is

$$\rho\frac{d\vec{v}}{dt} = -\vec{\nabla}p + \rho\vec{\nabla}\phi + \rho\nu\nabla^2\left(\vec{v} - \frac{\vec{v}}{K_1}\right) + \frac{\hbar^2\rho}{4m_e m_i}\vec{\nabla}\left(\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}\right) + 2(\vec{v}\times\vec{\Omega})\rho \tag{1}$$

The equation of continuity is given by

$$\varepsilon\frac{d\rho}{dt} + \rho\vec{\nabla}.\vec{v} = 0 \tag{2}$$

Poisson's equation for the gravitational potential is given as

$$\nabla^2\phi = -4\pi G\rho \tag{3}$$

The radiative heat transfer equation is given as

$$\frac{dT}{T} = (\Gamma - 1)\frac{d\rho}{\rho} \tag{4}$$

where $p = p_g + p_r$, $p_g = \rho RT$ gas pressure and $p_r = K_B T^4/3$ radiation pressure, ε, G, T, K_B, ρ, γ, ν, Ω, K_1, me, mi, $\hbar = \frac{h}{2\pi}$, c and Γ denotes the porosity of the medium, the gravitational constant, temperature, Boltzmann's constant, number density, ratio of two specific heat, kinematic viscosity, rotation , permeability, mass of electron, mass of ion, Planck's constant, velocity of sound and radiative heat functions, respectively.

It is assumed that the system follows adiabatic changes in an enclosure containing matter and radiation as given by Chandrasekhar

$$\Gamma = 1 + \frac{\Gamma_1 - b}{4 - 3b}, \qquad \Gamma_1 = b + \frac{(4-3b)^2(\gamma-1)}{b + 12(1-b)(\gamma-1)},$$

and $b = \frac{p_g}{p_g + p_r}$

If the radiation is negligible then $\Gamma = \Gamma_1 = \gamma$, while in case of $p_g \ll p_r$ is $\Gamma = \Gamma_1 = 4/3$.

3. LINEARIZED PERTURBATION EQUATIONS

In the linearization, we write the space and time dependent physical quantities p, ρ, $\vec{v}$, and ϕ, in the form of the sum of the equilibrium and perturbed part as

$$p = p_0 + \delta p,\ \rho = \rho_0 + \delta\rho\ ,$$
$$\phi = \phi_0 + \delta\phi, \quad \vec{v} = \vec{v}_o + \delta\vec{v}$$

The terms with subscript '0' denote the equilibrium part of the physical quantities. Perturbation in fluid velocity, fluid pressure, fluid density and gravitational potential are given as $\delta\vec{v}(0,0,v)$,δp, $\delta\rho$, $\delta\phi$ respectively.Using sum of the equilibrium and perturbed part of physical quantities in equations(1-4). We write the linearized perturbation equations of infinite homogeneous, viscous, rotating and infinite conducting gaseous molecular cloud of quantum plasmas, removing '0' from subscript in the quantities for simplicity. Thus, we obtain linearized perturbation equation of the considered system as

$$\frac{\partial\delta\vec{v}}{\partial t} = \frac{RT}{\rho}\frac{\partial\delta\rho}{\partial t} + R(1 + 4R_p)\frac{\partial\delta T}{\partial t} + \frac{\partial\delta\phi}{\partial \mathrm{t}} + \nu\nabla^2\left(\vec{v} - \frac{1}{K_1}\right) + \frac{\hbar^2}{4m_e m_i}\frac{\partial}{\partial \mathrm{z}}(\nabla^2\delta\rho) + 2(\delta\vec{v}\times\vec{\Omega}) \tag{5}$$

$$\frac{\partial\delta\rho}{\partial t} + \rho\frac{\partial\delta\vec{v}}{\partial z} = 0 \tag{6}$$

$$\frac{\partial^2\delta\phi}{\partial z^2} = -4\pi G\delta\rho \tag{7}$$

$$\frac{\partial\delta T}{\partial t} - (\Gamma - 1)\frac{T}{\rho}\frac{\partial\delta\rho}{\partial t} = 0 \tag{8}$$

4. DISPERSION RELATION AND DISCUSSION

Let us assume the perturbation of all the quantities very as

$$\exp\{-i\omega t + ikz\} \tag{9}$$

where ω is the frequency of harmonic disturbances, k is the wave number in z-direction. Combining equations (5) to (9) we get the dispersion relation as

$$\omega^2 - \frac{k^2}{RT}\left[1 + \left(1 + 4R_p\right)(\Gamma - 1)\right] + \omega\Omega_\nu + \frac{\hbar^2 k^4 \rho}{4 m_e m_i} - \Omega\mathrm{k} + 4\pi G\rho = 0 \tag{10}$$

where $\Omega_\nu = \nu\left(k^2 + \frac{1}{K_1}\right)$.

This dispersion relation shows the combined influence of rotation, quantum correction, kinematic viscosity and black body radiation on the Jeans instability of self-gravitating viscous infinite homogeneous infinite conducting and optically thick gaseous molecular cloud of quantum plasma. If we ignore the coefficients of viscosity, quantum correction and rotational frequency, then this dispersion relation reduces to Vranjes and Cadez[20]. Thus dispersion relation is modified by the presence of kinematic viscosity, rotational frequency and quantum correction. This dispersion relation will be able to predict the complete information about the stability and instability of disturbances propagating through infinite conducting and homogeneous rotating viscous gaseous molecular cloud plasma under the influence of black body radiation. The constant term of equation (10) has at least one positive root, this means that at least one value of ω is positive and this gives instability. The expression of the critical Jeans length is given as

$$\lambda_j \geq \lambda_c\left[\frac{1}{\gamma}\left\{1 + (\Gamma - 1)\left(1 + 4R_p\right) + \frac{\hbar^2}{4\lambda_c^2 m_e m_i} + \frac{\varepsilon\Omega^2}{\lambda_c}\right\}\right]^{1/2} \tag{11}$$

where λ_j is Jeans critical wave length and $\lambda_c = 2\pi/k$ is critical wave length.

From the condition of instability we find that jeans criterion id modified by the presence of parameters of porosity of the medium, rotation, black body radiation and quantum correction. The Jeans condition is affected by the presence of permeability and kinematic viscosity of the medium. Thus quantum correction and black body radiation stabilize the system. The porosity of the medium favour to destabilize the system. The kinematic viscosity and permeability of the medium always suppress the growth rate of unstable self-gravitating system.

5. CONCLUSION

In the present work, we have studied the problem of self-gravitational instability of a rotating viscous, optically thick plasma, considering the effects of permeability, porosity of the medium and black body radiation. The general dispersion relation is obtained which is modified due to presence of these parameters. We find that the Jeans condition remains valid but the expression of the critical Jeans wavelength is modified. The effect of viscosity and permeability are found to be stabilize the considered system. The porosity of the medium has destabilizing influence on the system.

REFERENCES

1. D. Pines, "Quantum plasma physics", *J. Nucl. Energy: Part C: Plasma Phys.*, 2 5 (1961).

2. D. Pines, "Elementary Excitations in solid", Westview Press, (1999).
3. J. H. Jeans, "Astronomy and Cosmogony", Cambridge Uni. Press, Cambridge (1929).
4. B. Bezzerides, D.F. DuBios, "Quantum electrodynamics of nonthermal relativistic plasmas: Kinetic theory", *Ann. Phys.* (N.Y.) 70, 10-66 (1972).
5. R. Hakim, J. Heyvaerts, "Covariant Wigner function approach for relativistic plasmas", *Phys. Rev. A* 18, 1250-1260 (1978).
6. P. K. Shukla and L. Stenflo "Jeans instability in quantum dusty plasmas", *Phys. Lett. A*, 355, 378-380 (2006).
7. W. Masood, M. Sallimullah and H. A. Shah "A quantum hydrodynamic model for multicomponent quantum magnetoplasma with Jeans term", *Phys. Lett. A*, 372, 6757-6760 (2008).
8. M. Sallimullah, M. Jamil, H. A. Shah and G. Murtaza "Jeans instability in a quantum dusty magnetoplasma", *Phys. Plasmas* , 16, 014502 (2009).
9. S. Chandrashekhar "Hydrodynamics and Hydromagnetic Stability" *Clarendon Press, Oxford* (1961).
10. B. Lehnert, "Magnetohydrodynamic waves under the action of the coriolis force", *Astrophys. J.*, 119, 647 (1954).
11. B. Lehnert, "Magnetohydrodynamic waves under the action of the coriolis force II", *Astrophys. J*., 121, 481 (1955).
12. R. K. Pensia, A. K. Patidar, V. Shrivastava, V. kumar and V. Prajapat, "The Role of Coriolis Force and Suspended Particles in the Fragmentation of Matter in the Central Region of Galaxy". *Journal of Bangladesh Academy of Sciences*, Vol. 36,(1), 1-12 (2012).
13. R. P. Prajapati, R. K. Pensia, S. Kaothekar, and R. K. Chhajlani, "Self-Gravitational Instability of Rotating Viscous Hall Plasma With Arbitrary Radiative Heat-Loss Functions And Electron Inertia", *Astrophys. Space Sci.* 327, 139-154 (2010).
14. A. K. Patidar, R. K. Pensia and V. Shrivastava, "Gravitational Instability of a Two Component Rotating Viscous Plasma under the Effect of Arbitrary Radiative Heat-Loss Functions and Electron Inertia". *J. Pure Appl. & Ind. Phys.* Vol.2 (3), 142-154 (2012).
15. R. K. Pensia, V. Ghorela and R. K. Chhajlani, "Thermal Instability of a Self Gravitating, Rotating Gaseous Plasma with Generalized Ohm's Law" *Ultra Scientist*, 20(3), 541-544 (2008).
16. R. K. Pensia, A. K. Patidar, V. Kumar and V. Prajapat, "The role of Heat-loss Function on the Growth of Molecular clouds in our Galaxy" *Ultra Scientist*, 23(1), 43-48 (2008).
17. M. P. Bora, & S. P. Talwar, "Magnetothermal Instability with Generalized Ohm's Law", *Phys. Fluids*, B5 (3), 950-955 (1993).
18. S. Kaothekar, & R. K. Chhajlani, "Self-gravitational instability of Partially Ionized plasma with Radiative Effects", *Journal of Physics: conference series*, 208, 012044 (2010).
19. J. H. Jeans "The Stability of Spherical Nebula" *Phil. Trans. Roy. Soc. London*, 199, p 1-53 (1902).
20. J. Vranjes and V. Cadez, "Gravitational Instability of a Homogeneous Gas cloud With Radiation, *Astrophys. Space Sci.*, 164,329 (1990).